Universal Post-quench Dynamics
at a Quantum Critical Point

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Rutgers University, 10 March 2016

References:
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Why probing in non-equilibrium?

- Give access to excitations and their relaxation
  - Inelastic scattering (neutron, resonant x-ray) probes dynamic response in equilibrium
  - Pump-probe spectroscopic techniques probe non-equilibrium response
- Measure fluctuations arising from nearby competing phases

Primary interest and puzzle of correlated materials often lies in properties of excited states.

Examples:
- Linear in temperature resistivity in cuprates and heavy-fermions
- Fractionalized excitations in spin liquids (e.g. $\alpha$-RuCl$_3$)
Universality at classical and quantum criticality

Materials:
- Rare-earth magnetic insulators
- Heavy-fermion compounds
- Unconventional superconductors
- 2D electron gases

Universal behavior:
- Divergent correlation length and time
  \[ \xi \propto \delta r^{-\nu} \text{ and } \xi_T \propto \delta r^{-\nu z} \]
- Power-laws, critical exponents
- Data collapse due to scaling
- Precise experiment-theory comparison


Universality at classical and quantum criticality

Materials:
- Rare-earth magnetic insulators
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Universal behavior:
- Divergent correlation length and time
  \( \xi \propto \delta r^{-\nu} \) and \( \xi_\tau \propto \delta r^{-\nu z} \)
- Power-laws, critical exponents
- Data collapse due to scaling
- Precise experiment-theory comparison

From [4]

Universality comes with potential for quantitative predictions for strongly interacting systems far from equilibrium.

Example: \( N \)-component \( \varphi^4 \) field theory

- \( N=1 \): Ising model in transverse field (CoNbO\(_6\))
- \( N=2 \): Sc-insulator QPT (XY model)
- \( N=3 \): quantum dimer systems (TICuCl\(_3\))

\[
S = \frac{1}{2} \int d^d x \int_0^{1/T} d\tau \left( (\partial_\tau \varphi)^2 + c^2 (\nabla \varphi)^2 + r_0 \varphi^2 + \frac{u}{2} (\varphi^2)^2 \right)
\]

Here: \( z=1 \)

Universality in equilibrium

Scaling of magnetization in equilibrium [1]

\[ m(\delta r, h) = b^{-\beta/\nu} m(b^{1/\nu} \delta r, b^{\delta \beta/\nu} h) \]

\[ m(\delta r, 0) \propto \delta r^{\beta} \]
\[ m(0, h) \propto h^{1/\delta} \]

Data collapses onto universal curve:

\[ m(\delta r, h) = \delta r^{\beta} \Phi_m(h/\delta r^{\beta \delta}) \]

Critical exponents define universality class:
- Depends only on dimensionality and symmetry
- Calculate exponents using the renormalization group in small \( \epsilon = d_{up} - d \) or \( 1/N \)

Theoretical prediction from \( \epsilon \)-expansion [2]:

\[ \gamma = 1.40, \beta = 0.38, \delta = 4.68 \]

---

Universality and scaling in non-equilibrium

Does universality occur also in non-equilibrium situations?

Is universality in non-equilibrium characterized by new critical exponents?
Universality and scaling in non-equilibrium

Does universality occur also in non-equilibrium situations?

YES!

- Near-equilibrium dynamics and long-time approach to equilibrium described by power laws with equilibrium exponents [1]
- Kibble-Zurek mechanism describing defect formation in parameter sweeps through critical points [2-6]

Universality and scaling in non-equilibrium

Does universality occur also in non-equilibrium situations?

Example (Cheong group): Thermal Kibble-Zurek quench in hexagonal manganites $\text{RMnO}_3$ with $R = \text{Sc, Y, Dy, Lu}$

From [4]

From [5]

From [4]

Universality and scaling in non-equilibrium

Does universality occur also in non-equilibrium situations?

YES!

Is universality in non-equilibrium characterized by new critical exponents?

YES, sometimes! Topic of today’s talk

Can use non-equilibrium dynamics as a new tool to study quantum critical materials.

General protocol of a quench

- Bring system to non-equilibrium by rapid change of parameter
  - Strain [1], magnetic field [2]: correlated materials
  - Temperature [3, 4]: ferroelectric materials RMnO$_3$
  - Laser intensity [5, 6, 7]: cold-atom setups

- Theoretically well-defined protocol
  - Prepare system in ground state of initial Hamiltonian
  - Perform unitary time evolution with a different Hamiltonian

\[
|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle
\]

Non-equilibrium dynamics: \(\mathcal{O}(t) = \langle \Psi(t)|\hat{\mathcal{O}}|\Psi(t)\rangle\)

Model and quench protocol

Hamiltonian: N-component $\varphi^4$-field theory

$$H_s(t) = \frac{1}{2} \int d^d x \left( \pi^2 + (\nabla \varphi)^2 + r_0(t) \varphi^2 + \frac{u(t)}{2N} (\varphi \cdot \varphi)^2 - h(t) \varphi \right)$$

Equilibration? At finite temperature?

Sudden quench protocol (fast KZ sweep)

$$r_0(t) = r_{0,i} + \theta(t)(r_{0,c} - r_{0,i})$$

Quench deposits energy
Model and quench protocol

Hamiltonian: N-component $\varphi^4$-field theory coupled to a bath

$$H_s(t) = \frac{1}{2} \int d^d x \left( \pi^2 + (\nabla \varphi)^2 + r_0(t)\varphi^2 + \frac{u(t)}{2N}(\varphi \cdot \varphi)^2 - h(t)\varphi \right)$$

$$H_{sb} = \frac{1}{2} \sum_j \int d^d x X_j \cdot \varphi$$

$$H_b = \frac{1}{2} \sum_j \int d^d x \left( P_j^2 + \Omega_j^2 X_j^2 \right)$$

Bath spectral function:

$$\text{Im} \eta(\omega) = \gamma \omega |\omega|^{-1+2/z} e^{-|\omega|/\omega_c}$$

Induces dissipation, dynamic exponent $z > 1$

Bath ensures equilibration at $T = 0$ at long times.

Quench deposits energy

Model and quench protocol

Hamiltonian: N-component $\varphi^4$-field theory coupled to a bath

$$H_s(t) = \frac{1}{2} \int d^d x \left( \pi^2 + (\nabla \varphi)^2 + r_0(t) \varphi^2 + \frac{u(t)}{2N} (\varphi \cdot \varphi)^2 - h(t) \varphi \right)$$

$$H_{sb} = \frac{1}{2} \sum_j \int d^d x \mathbf{X}_j \cdot \varphi$$

$$H_b = \frac{1}{2} \sum_j \int d^d x \left( \mathbf{P}_j^2 + \Omega_j^2 \mathbf{X}_j^2 \right)$$

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$$\text{Im} \eta(\omega) = \gamma \omega \left| \omega \right|^{-1+2/z} e^{-|\omega|/\omega_c}$$

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Quench deposits energy

$\langle \varphi \rangle \neq 0$

$\langle \varphi \rangle = 0$

Main results and scaling analysis
Scaling in non-equilibrium: order parameter

- Universal dynamics of the order parameter

\[ m(\delta r_i, \delta r_f, t) = b^{-\beta/\nu} m(b^\kappa/\nu \delta r_i, b^{1/\nu} \delta r_f, b^{-z} t) \]

Quench right to the critical point

\[ m(\delta r_i, 0, t) = t^{-\beta/(\nu z)} \Phi(t^\kappa/(\nu z) \delta r_i) \]


Scaling in non-equilibrium: order parameter

Quench right to the critical point

\[ m(\delta r_i, 0, t) = t^{-\beta/(\nu z)} \Phi(t^\kappa/(\nu z) \delta r_i) \]

Long times: \( \Phi(y \gg 1) = \text{const.} \)

\[ m(\delta r_i, 0, t) \propto t^{-\beta/(\nu z)} \]

Long-time approach to equilibrium is described by equilibrium scaling exponents.
Scaling in non-equilibrium: order parameter

Quench right to the critical point

\[ m(\delta r_i, 0, t) = t^{-\beta/(\nu z)} \Phi(t^\kappa/(\nu z) \delta r_i) \]

Long times: \( m(\delta r_i, 0, t) \propto t^{-\beta/(\nu z)} \)

Short times: \( \Phi(y < 1) = y^\beta \)

\[ m(\delta r_i, 0, t) \propto t^{\beta(\kappa-1)/(\nu z)} = t^\theta \]

Potential for a new dynamical exponent.

Scaling in non-equilibrium: order parameter

Quench right to the critical point

\[ m(\delta r_i, 0, t) = t^{-\beta/(\nu z)} \Phi(t^{\kappa/(\nu z)} \delta r_i) \]

Long times: \( m(\delta r_i, 0, t) \propto t^{-\beta/(\nu z)} \)

Short times: \( \Phi(y \ll 1) = y^\beta \)

\[ m(\delta r_i, 0, t) \propto t^{\beta(\kappa-1)/(\nu z)} = t^\theta \]

Potential for a new dynamical exponent.

Crossover timescale:

\[ t^* = \delta r_i^{-\nu z/\kappa} \]

Damping sets beginning of universal regime

\[ t_\gamma \propto \gamma^{-z/(2(z-1))} \]

Scaling in non-equilibrium: correlation length

- Correlation length becomes time-dependent. At critical point we find
  \[ \xi(t) \propto t^{1/z} \]

- Rapid quench first leads to a non-universal collapse of the correlation length [1, 2]
- Then, dynamic build-up of correlations

Scaling in non-equilibrium: correlation functions

- Dynamic scaling of correlation and response functions

In equilibrium:

\[ \{ \varphi_{eq}(q, t), \varphi_{eq}(q, t') \} = G^K_{eq}(q, t - t') \]

Non-equilibrium: now depends in general on both time variables \( t \) and \( t' \)

\[ \{ \varphi(q, t), \varphi(q, t') \} = \left( \frac{t}{t'} \right)^\theta \frac{F \left( \frac{q^z t / \gamma^z / 2}{t / t'} \right)}{q^{2-z} \gamma^z / 2} \]

Singular dependence captured by new exponent \( \theta \)

Smooth scaling function \( F \)
Cold-atom experimental realization of quench to quantum critical point
Dynamic scaling after quench in cold-atom gas

- Two-component 1D degenerate Bose gas = spin gas
- Miscible-Immiscible quantum phase transition

Hamiltonian:

\[ H = \int \frac{\rho}{2} \left( |\partial_x S|^2 + \Omega S^x - \Omega_c (S^z)^2 \right) \]

- Rabi coupling \( \propto \) laser intensity
- Single-ion anisotropy \( \propto \alpha_{\uparrow\downarrow} \)
- Heisenberg exchange from kinetic energy

Density difference = Spin z-component

\[ S^z = (\rho_{\uparrow} - \rho_{\downarrow})/\rho \]
Miscible-Immiscible quantum phase transition

- Two-component 1D degenerate Bose gas = spin gas
  \[ S^\alpha = \rho^{-1} b^\dagger_\tau \sigma^\alpha_\tau, b_\tau \]
- Miscible-Immiscible quantum phase transition

Hamiltonian:

\[ H = \int_x \frac{\rho}{2} \left( |\partial_x S|^2 + \Omega S^x - \Omega_c (S^z)^2 \right) \]

Rabi coupling \( \propto \) laser intensity

Single-ion anisotropy \( \propto a_{\uparrow\downarrow} \)

Miscible = Paramagnetic
Immiscible = Ferromagnetic

Ising quantum phase transition

Miscible

Immiscible

\( S^z = (\rho_{\uparrow} - \rho_{\downarrow})/\rho \)
Quench of Rabi coupling

- Sudden quench from paramagnetic state towards critical point
- Measure spin-spin correlation function to extract correlation length $\xi$

Correlation length increases as $\xi(t, 0) \propto t^{1/z}$

Here: $z = 1$ (mean-field result)

Reason: technical limitation to come close enough to critical point to reach critical regime
Scaling of correlation function at fixed time

- Data collapse of correlation function at long times when rescaling lengths with $\xi_{eq}$

- Equilibrium correlation length scales as

$$\xi(\delta r_f) \propto \delta r_f^{-\nu}$$

Here: $\nu = \frac{1}{2}$ (mean-field result)

- Interaction effects only visible for quenches closer to critical point

- Short-time scaling could be observed in non-equal time correlation functions and when quenching out of ordered phase
Dynamic scaling after rapid quench to quantum criticality
Quench in non-interacting model coupled to bath

- Post-quench retarded Green’s functions for $u=0$ in presence of bath

Heisenberg equations of motion

$$
\left( \partial_t^2 + r_{0,f} + q^2 \right) \varphi(q,t) = \int_0^\infty ds \eta(t-s) \varphi(q,s) + \Xi(q,t) + h(q,t)
$$

Source operator depends on bath initial states

$$
\Xi(q,t) = - \sum_j c_j \left( X_j^0(q) \cos(\Omega_j t) + \frac{1}{\Omega_j} P_j^0(q) \sin(\Omega_j t) \right)
$$
Quench in non-interacting model coupled to bath

- Post-quench retarded Green’s functions for \( u=0 \) in presence of bath

Heisenberg equations of motion

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(\partial_t^2 + r_{0,f} + q^2) \varphi(q, t) = \int_0^\infty ds \eta(t-s) \varphi(q, s) + \Xi(q, t) + h(q, t)
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\]

Solve via Laplace transformation

\[
\varphi(q, \omega) = \int_0^\infty dt e^{i(\omega+i0^+)t} \varphi(q, t)
\]

\[
\varphi(q, \omega) = F(q, \omega) g_f^R(q, \omega)
\]

Force operator

\[
F(q, \omega) = \pi_0(q) - i\omega \varphi_0(q) + \Xi(q, \omega) + h(q, \omega)
\]

Bare retarded post-quench Green’s function

\[
g_f^R(q, \omega) = \frac{1}{\omega^2 - r_{0,f} - q^2 + \eta(\omega)}
\]
Bare post-quench Keldysh Green’s function

- Find $G$ via commutators of $\varphi (q, \omega) = F(q, \omega) g_f^R(q, \omega)$

**Retarded:**

$$G^R(t, t') = -i \theta(t-t') \langle [\varphi_H(t), \varphi_H(t')] \rangle$$

Depends on $(t-t')$ only (no longer the case for $u > 0$)

$$g_f^R(q, \omega) = \frac{1}{\omega^2 - r_{0,f} - q^2 + \eta(\omega)}$$

**Correlation:**

$$G^K(t, t') = -i \langle \{\varphi_H(t), \varphi_H(t')\} \rangle$$

Depends on both $t$ and $t'$. Use double Laplace transform

$$g_f^K(q, \omega, \omega') = M(q, \omega, \omega') g_f^R(q, \omega) g_f^R(q, \omega')$$

Memory function $M$ depends on initial conditions:

$$M(q, \omega, \omega') = i \frac{g_i^K(q, \omega) + g_i^K(q, \omega')}{\omega + \omega' + i0^+} g_i^R(q, \omega)^{-1} g_i^R(q, \omega')^{-1}$$
Bare post-quench Keldysh Green’s function

Exponential decay to equilibrium at equal times and $u=0$.

$$g^K_f(q, t, t) = \frac{f^K_0(q^{z}t/\gamma^{z/2}, 1)}{q^{2-\eta-z\gamma^{z/2}}}$$

For scaling limit of large initial mass $\delta r_i$.

- Overdamped for $z > 2$: sub-Ohmic
- Underdamped for $z \leq 2$: (super)-Ohmic
Equal time correlations in presence of interactions

- Free Keldysh function approaches equilibrium exponentially
- Interacting Keldysh function exhibits power-law decay
  - Amplitude depends on universal exponent $\theta$

$$G^K_T(q, t, t) = G^K_{eq}(q) - \frac{f(\gamma, z)}{q^{4-z}} \frac{\theta}{t^{2/z}}$$

Critical fluctuations significantly slow down equilibration.

with $f(\gamma, z) = \frac{2\Gamma(2/z)}{c_K \sin(\pi/z)} \frac{1}{\gamma}$ and coefficient: $c_K = \frac{4 \sin(\pi z/2)}{z(2-z) \sin^{z/2}(\pi/z)}$
Distribution Wigner function at long times

At long times it holds

\[ G^K_r(q, t_a, \omega) = 2i \coth \left( \frac{\omega}{2T} \right) \left[ 1 + 2r(t_a) \text{Re} G^{R}_{eq}(q, \omega) \right] \text{Im} G^{R}_r(q, t_a, \omega) \]

Introduce time-dep. distribution function \( n(t_a, \omega) = n_B(\omega) + \delta n(t_a, \omega) \)

Deviation from equilibrium

\[
\delta n(t, \omega) = - \coth \left( \frac{\omega}{2T} \right) \frac{\gamma \theta \Gamma(2/z)}{\sin(\pi/z) t^{2/z}} \text{Re} G^{R}_{eq}(q, \omega)
\]

- Non-thermal since algebraically decaying at large frequencies \( \delta n \propto |\omega|^{-2/z} \)
- Slow approach to equilibrium described by power-law \( \delta n \propto t_a^{-2/z} \)
- Can change sign: density matrix non-diagonal in energy basis (coherence)
Solving the (non-)equilibrium large-N equations
Interactions in large-N approximation

- Pre-quench equilibrium large-N equations

\[ h_i = r_i \phi_i \]
\[ r_i = \bar{r}_{0,i} + \frac{u_i}{2} \phi_i^2 + u_i \int_{q, \omega_n} G_{r_i}^M (q, \omega_n) \]

Matsubara Green’s function

\[ G_{r_i}^M (q, \omega_n) = \frac{-1}{\omega_n^2 + r_i + q^2 - \delta \eta^M (\omega_n)} \]

Bath induced self-energy

\[ \delta \eta^M (\omega_n) = -\frac{\gamma}{\sin \frac{\pi \alpha}{2}} |\omega_n|^\alpha \]

Dynamic critical exponent \( z = 2/\alpha \)

Initial distance to QCP (in presence of bath and interactions)

Pre-quench order parameter value
Interactions in large-N approximation

- Pre-quench equilibrium large-N equations

\[
\begin{align*}
h_i &= r_i \phi_i \\
r_i &= \bar{r}_{0,i} + \frac{u_i}{2} \phi_i^2 + u_i \int_{q,\omega_n} G_{r_i}^M (q, \omega_n)
\end{align*}
\]

Ordered phase: \( \phi_i \neq 0 \Rightarrow r_i(h_i = 0) = 0 \)  Massless spectrum (in 1/N)

Phase transition when \( \phi_i = 0 \) and \( r_i = 0 \)

\[
\bar{r}_{0,c} = -u \int_{q,\omega}^{\Lambda} \frac{1}{\omega^2 + q^2 + \delta \eta^M (\omega)}
\]

Universality as function of

\[
\delta r_i = \bar{r}_{0,i} - r_{0,c}
\]

For example: \( \phi_i \propto (-\delta r_i)^\beta \), \( \xi = r_i^{-1/2} \propto \delta r_i^{-\nu} \) with \( \beta = 1/2, \nu = d + z - 2 \)
Post-quench large-N equations

- Non-equilibrium large-N equations:
  - Time-dependent mass $r_i \rightarrow r(t)$
  - Time-dependent order parameter $\phi_i \rightarrow \phi(t)$
  - Self-energy given by Keldysh Green’s function

$$\int_{q,\omega_n} G^M_{ri} \rightarrow \int_{q,t} G^K_r(q,t,t)$$

\[
\begin{align*}
    h_f &= -\int_0^\infty dt' (G^R_r)^{-1}(t,t') \phi(t') - \phi_i \int_{-\infty}^0 dt' \delta \eta(t-t') \\
    r(t) &= \bar{r}_{0,f} + \frac{uf}{2} \phi^2(t) + \frac{uf}{2} \int_q iG^K_r(q,t,t) \ .
\end{align*}
\]

Retarded Green’s function contains \textit{time-dependent mass} as well

\[
(G^R_r)^{-1}(t,t') = -\left(\partial_t^2 + r(t) - \nabla^2\right) \delta(t-t') + \delta \eta(t-t')
\]
Quench from disordered phase to critical point

- Initial magnetization vanishes $\phi_i = 0 \Rightarrow \phi(t) = 0$
- Quench right to critical point $\bar{r}_{0,f} = 0$

$$r(t) = \frac{u}{2} \int \frac{d^d k}{(2\pi)^d} \left( iG_r^K(k,t,t) - iG_{eq}^K(k) \right)$$

![Diagram](image-url)
Quench from disordered phase to critical point

- Initial magnetization vanishes $\phi_i = 0 \Rightarrow \phi(t) = 0$
- Quench right to critical point $\tilde{r}_{0,f} = 0$

\[ r(t) = \frac{u}{2} \int \frac{d^d k}{(2\pi)^d} \left( iG^K_r(k,t,t) - iG^K_{eq}(k) \right) \]

- Ansatz for mass term (that provides self-consistent solution)

\[ r(t) = \frac{\gamma a}{t^{2/z}} \]

Light-cone amplitude

Light-cone dynamical growth of correlation length

\[ \xi(t) = r(t)^{-1/2} \propto t^{1/z} \]
Consequence of dynamic mass $r(t)$

- Logarithmic divergencies at leading order ($t' \ll t$)

$$
\delta G^R(k, t, t') = \int_{t\gamma}^{t} ds g^R(k, t-s) \frac{\gamma a}{s^{2/z}} g^R(k, s-t')
$$

- Bare Green’s function completely local at short times $(\sqrt{\gamma}/q)^z > t, t' > t\gamma$

$$
g^R(q, \omega) \approx \frac{1}{\delta \eta(\omega)} \quad \Rightarrow \quad g^R(q, t) \approx -\frac{\sin(\pi/z)}{\gamma \Gamma(2/z)} t^{2/z-1}
$$

Justification for deep-quench limit: **locality corresponds to small correlation length directly after the quench.**
Consequence of dynamic mass $r(t)$

- Logarithmic divergencies at leading order $(t' \ll t)$

\[
\delta G^R(k, t, t') = \int_{t'}^t ds g^R(k, t - s) \frac{\gamma a}{s^{2/z}} g^R(k, s - t')
\]

\[
= -\frac{a \sin(\pi/z)}{\gamma \Gamma(2/z)} g^R(t - t') \log(t/t')
\]

New non-equilibrium critical exponent

\[
\theta = -\frac{a \sin(\pi/z)}{\gamma \Gamma(2/z)}
\]

Determined by light-cone amplitude $a$.

Scaling form of retarded Green’s function

\[
G^R(k, t, t') = \left( \frac{t}{t'} \right)^\theta \frac{f^R(k^{z\gamma/2}t / \gamma, t'/t)}{k^{2-n-z\gamma/2}}
\]
Consequence of dynamic mass $r(t)$

- Logarithmic divergencies at leading order ($t' \ll t$)

\[
\delta G^R(k, t, t') = \int_{t'}^t ds g^R(k, t - s) \frac{\gamma a}{s^{2/\nu}} g^R(k, s - t')
= -\frac{a \sin(\pi/\nu)}{\gamma \Gamma(2/\nu)} g^R(t - t') \log(t/t')
\]

New non-equilibrium critical exponent

\[
\theta = -\frac{a \sin(\pi/\nu)}{\gamma \Gamma(2/\nu)}
\]

For Ohmic bath: $\theta = \frac{\epsilon}{4}$
Determining light-cone amplitude $a$

- Crucial: interacting Keldysh function decays as power-law at long times

\[
G^K_r (q, t, t) = G^K_{\text{eq}} (q) + \frac{2r(t)}{c_K q^{4-z} \gamma z/2}
\]

Critical fluctuations slow down equilibration.

Inserting into self-consistency equation: $G^K_r = g^K + G^K_1$

\[
r(t) = \frac{uK_d}{2} \int_0^\Lambda dq \, q^{3-z-\epsilon} \left[ iG^K_r (q, t, t) - iG^K_{\text{eq}} (q) \right]
\]

Yields:

\[
\frac{a \gamma}{t^{2/z}} = \frac{uK_d}{2z \gamma^{z/2}} \frac{\gamma C_0 t^{\epsilon/z}}{\gamma^{\epsilon/z} t^{2/z}} + \frac{ua \gamma K_d}{c_K \epsilon \gamma^{z/2} t^{2/z}} \left( \Lambda^{-\epsilon} - \frac{t^{\epsilon/z}}{\gamma^{\epsilon/2}} \right)
\]

Exponent:

\[
a = \frac{c_K C_0}{2z} \epsilon
\]

Solve numerically for general $z$

\[
C_0 = i \int_0^\infty dx \, x^{\frac{2}{z}-1} \left( f^K (x, 1) - F^K_{\text{eq}} \right)
\]

Interaction fixed-point:

\[
u = u^* \equiv \frac{c_K \gamma^{z/2} \Lambda^\epsilon}{K_d} \epsilon
\]
Order parameter dynamics

Two different universal time regimes

- Short time \( m(\delta r_i, 0, t) \propto t^{\theta} \) for \( t < t^* \propto \delta r_i^{-\nu z / \kappa} \)
- Long time \( m(\delta r_i, 0, t) \propto t^{-\beta / (\nu z)} \) for \( t > t^* \)

\[ \theta > 0 \] Ohmic and sub-Ohmic

New short time critical exponent depends on \( z \)
- Recovery of magnetization due to fast growing correlation length
- Depends on dynamic critical exponent \( z \)
- Test hyperscaling, since \( \theta \) vanishes in mean-field
Order parameter dynamics

Two different universal time regimes

- Short time \( m(\delta r_i, 0, t) \propto t^\theta \) for \( t < t^* \propto \delta r_i^{-\nu z/\kappa} \)
- Long time \( m(\delta r_i, 0, t) \propto t^{-\beta/(\nu z)} \) for \( t > t^* \)

\[ \theta < 0 \] super-Ohmic

New short time critical exponent depends on \( z \)

- Recovery of magnetization due to fast growing correlation length
- Depends on dynamic critical exponent \( z \)
- Test hyperscaling, since \( \theta \) vanishes in mean-field
Summary and Outlook

- Quench to quantum critical points results in universal post-quench dynamics
- Characterized by a new critical exponent
- Correlation length collapses after quench and recovers in a light-cone fashion

References:

Quench in closed system
- Coupled bosonic order parameters, e.g. competition between superconductivity and magnetism.
- Fermionic field theory (metallic magnets, graphene)
- Propagation of entanglement

Thank you for your attention.